$$
\begin{array}{r}
AS \\
+\quad A \\
\hline
MOM
\end{array}
$$

NOTATION:
C_K = carry over to column k

1) $C_3 = 1 \therefore M = 1$

2) $A \neq O \therefore C_2 = 1$

$$
\begin{array}{rrr}
1 & 1 & \\
 & A & S \\
+ & & A \\
\hline
M & O & M \\
1 & & 1
\end{array}
$$

3) $1 + A = O + 10$

$A = O + 9$

$\therefore O = 0, A = 9$

$$
\begin{array}{rrr}
1 & 1 & \\
 & A & S \\
 & 9 & \\
+ & & A \\
 & & 9 \\
\hline
M & O & M \\
1 & 0 & 1
\end{array}
$$

4) $S + 9 = 1 + 10$

$S = 11 - 9 = 2$

SOL'N:

$$
\begin{array}{r}
92 \\
+\quad 9 \\
\hline
101
\end{array}
$$

$$\begin{array}{cccccc} & & O & O & O & H \\ + & & F & O & O & D \\ \hline & F & I & G & H & T \end{array}$$

1) $C_5 = 1$; $\therefore F = 1$

2) If $C_4 = 0$, then

$O + 1 = I + 10$

$O = I + 9$

$\therefore I = 0$ and $O = 9$

Impossible! This contradicts $O + O = 18$ and $C_4 = 0$.

$\therefore C_4 = 1$ and so

$O + 2 = I + 10$

$O = I + 8$

$\therefore I = 0$ $(I \neq 1)$

and $O = 8$

$$\begin{array}{cccccc} & 1 & 1 & 1 & & \\ & & O & O & O & H \\ & & 8 & 8 & 8 & \\ + & & F & O & O & D \\ & & 1 & 8 & 8 & \\ \hline & F & I & G & H & T \\ & 1 & 0 & & & \end{array}$$

3) $G = 7$

4) $C_2 = 0$ or 1

$C_2 = 1 \Rightarrow H = 7$ impossible

$\therefore C_2 = 0$ and $H = 6$

6) $6 + D = T$

$\Rightarrow D = 0, 1, 2$ or 3

But, $I = 0$, $F = 1$ and $O = 8$

$\therefore D = 3$ and $T = 6 + 3 = 9$

SOLUTION:

$$\begin{array}{cccccc} & & 8 & 8 & 8 & 6 \\ + & & 1 & 8 & 8 & 3 \\ \hline & 1 & 0 & 7 & 6 & 9 \end{array}$$

$$\begin{array}{rccccc} & C & H & E & C & K \\ + & & & T & H & E \\ \hline & T & I & R & E & S \end{array}$$

1) Since $C \neq T$ and $H \neq I$ there are two "carry ones"

$\therefore$ $C_4 = 1$ and $C_5 = 1$

$$\begin{array}{rccccc} & 1 & 1 & & & \\ & C & H & E & C & K \\ & & & T & H & E \\ \hline & T & I & R & E & S \end{array}$$

2) $\boxed{C + 1 = T}$

$H + 1 = I + 10$

$H = I + 9$

$\therefore$ $I = 0$, $H = 9$

$$\begin{array}{rccccc} & 1 & 1 & & & \\ & C & H & E & C & K \\ & & 9 & & & \\ & & & T & H & E \\ & & & & 9 & \\ \hline & T & I & R & E & S \\ & & 0 & & & \end{array}$$

3) Column 2:

$C + 9 = E$ impossible

$\therefore$ $C + 9 = E + 10$

$\boxed{C = E + 1}$ and $C_3 = 1$

4) E, C, T are consecutive integers

5)

$$\begin{array}{rccccc} & 1 & 1 & 1 & & \\ & C & H & E & C & K \\ & & 9 & & & \\ & & & T & H & E \\ + & & & & 9 & \\ \hline & T & I & R & E & S \\ & & 0 & & & \end{array}$$

$1 + E + T = R + 10$

$E + T = R + 9$

Since $R \neq 0$, $E + T \geq 10$

Can discard

$E = 1$, $C = 2$, $T = 3$

$E = 2$, $C = 3$, $T = 4$

$E = 3$, $C = 4$, $T = 5$

6) Generate & check:

H	I	E	C	T	K	R	S
9	0	4	5	6	*	*	*

$$\begin{array}{rccccc} & 1 & 1 & 1 & & \\ & C & H & E & C & K \\ & 5 & 9 & 4 & 5 & \\ & & & T & H & E \\ + & & & 6 & 9 & 4 \\ \hline & T & I & R & E & S \\ & 6 & 0 & & 4 & \end{array}$$

column 3: $R = 1$

column 2: $C_2 = 0$

$\therefore$ $K + 4 = S$

Since $K \neq 0, 1, 2$,

$K = 3$ and $S = 7$

SOL'N:

$$\begin{array}{rccccc} & 5 & 9 & 4 & 5 & 3 \\ + & & & 6 & 9 & 4 \\ \hline & 6 & 0 & 1 & 4 & 7 \end{array}$$

$$\begin{array}{rccccc} & & S & E & N & D \\ + & & M & O & R & E \\ \hline & M & O & N & E & Y \end{array}$$

1) $C_5 = 1 \therefore M = 1$

$$\begin{array}{rccccc} & & {}^{1}S & E & N & D \\ + & & \underset{1}{M} & O & R & E \\ \hline & \underset{1}{M} & O & N & E & Y \end{array}$$

2) $C_4 + S + 1 = O + 10$

$C_4 + S = O + 9$

Case I: $C_4 = 0$

$S = O + 9$

$\therefore O = 0, S = 9$

Case II: $C_4 = 1$

$S = O + 8$

$\therefore O = 0, S = 8$

$\therefore O = 0$ and $S = 8$ or 9

3) $C_3 + E + O = N$

$C_3 + E = N$

$\therefore C_3 = 1$ and $\boxed{N = E + 1}$

OR $C_3 + E + O = N + 10$

$C_3 + E = N + 10$

Case I: $C_3 = 0 \Rightarrow E = N + 10$ impossible

Case II: $C_3 = 1 \Rightarrow E = N + 9$ impossible

$\therefore C_3 = 1, C_4 = 0$ and $\boxed{N = E + 1}$

$\therefore S = 9$

$$\begin{array}{rccccc} & 1 & \overset{1}{\underset{9}{S}} & E & N & D \\ + & & \underset{1}{M} & \underset{0}{O} & R & E \\ \hline & \underset{1}{M} & \underset{0}{O} & N & E & Y \end{array}$$

4) column 2: $C_2 + N + R = E$

$C_2 + (E+1) + R = E$

$C_2 + 1 + R = 0$
impossible

OR $C_2 + N + R = E + 10$

$C_2 + (E+1) + R = E + 10$

$C_2 + R = 9$

Case I: $C_2 = 0 \Rightarrow R = 9$
impossible

Case II: $C_2 = 1 \Rightarrow R = 8$

$\therefore R = 8$

$$\begin{array}{rccccc} & 1 & \underset{9}{S} & \overset{1}{E} & \overset{1}{N} & D \\ + & & \underset{1}{M} & \underset{0}{O} & \underset{8}{R} & E \\ \hline & \underset{1}{M} & O & N & E & Y \end{array}$$

5) $Y \geq 2 \Rightarrow D + E \geq 12$

Unassigned digits: 2, 3, 4, 5, 6, 7

Possible (D, E) pairs:

$(5,7), (7,5), (6,7), (7,6)$

$E = 5 \Rightarrow N = 6$

$E = 6 \Rightarrow N = 7$ impossible $(D = 7)$

$E = 7 \Rightarrow N = 8$ impossible $(R = 8)$

$\therefore E = 5, N = 6, D = 7$

and $Y = 2$

SOL'N:

$$\begin{array}{rccccc} & & 9 & 5 & 6 & 7 \\ + & & 1 & 0 & 8 & 5 \\ \hline & 1 & 0 & 6 & 5 & 2 \end{array}$$

$$\begin{array}{rccccccc}
 & & & F & I & F & T & Y \\
+ & & S & T & A & T & E & S \\
\hline
 & A & M & E & R & I & C & A
\end{array}$$

1) $C_7 = 1 \quad \therefore A = 1$

2) $S \neq M \quad \therefore C_6 = 1$

$S + 1 = M + 10$

$S = M + 9$

$\therefore M = 0, S = 9$

$Y + 9 = 1 + 10$

$Y = 11 - 9 = 2$

$$\begin{array}{rccccccc}
 & 1 & 1 & & & & 1 & \\
 & & & F & I & F & T & Y \\
 & & & & & & & 2 \\
 & & S & T & A & T & E & S \\
+ & & 9 & & 1 & & & 9 \\
\hline
 & A & M & E & R & I & C & A \\
 & 1 & 0 & & & & & 1
\end{array}$$

3) $\boxed{F+T = E+10}$

4) $I + 1 + C_4 = R$

OR $I + 1 + C_4 = R + 10$

<u>Case 1</u>: $I+1+C_4 = R+10$

$I + C_4 = R + 9$

$C_4 = 0 \Rightarrow I = R + 9$

$\Rightarrow R = 0, I = 9$ impossible

$C_4 = 1 \Rightarrow I = R + 8$

$\left.\begin{array}{l} \Rightarrow R=0, I=8 \\ \text{or } R=1, I=9 \end{array}\right\}$ impossible

$\therefore I + 1 + C_4 = R$ and $C_5 = 0$

<u>Case 2</u>: $I + 1 + C_4 = R$

$C_4 = 0 \Rightarrow I + 1 = R$

$C_4 = 1 \Rightarrow I + 2 = R$

5) column 3: $F+T+C_3 = I$ or $F+T+C_3 = I+10$

<u>Case 1</u>: $F+T+C_3 = I$

But, $F+T = E+10 \Rightarrow E+10+C_3 = I$ impossible

$\therefore F+T+C_3 = I+10$ and $C_4 = 1$

$\therefore \boxed{I+2=R}$

<u>Case 2</u>: $F+T+C_3 = I+10$

$C_3 = 0 \Rightarrow F+T = I+10$

But, $F+T = E+10 \Rightarrow I = E$ impossible

$\therefore C_3 = 1$ and $\boxed{F+T = I+9}$

$$\begin{array}{rccccccc}
 & 1 & 1 & & 1 & 1 & 1 & \\
 & & & F & I & F & T & Y \\
 & & & & & & & 2 \\
 & & S & T & A & T & E & S \\
+ & & 9 & & 1 & & & 9 \\
\hline
 & A & M & E & R & I & C & A \\
 & 1 & 0 & & & & & 1
\end{array}$$

6) $F+T = 10+E$ and $F+T = I+9$

$\Rightarrow \boxed{I = E+1}$

7) $T+E+1 = C+10 \Rightarrow \boxed{T+E = C+9}$

8) <u>Generate and Test</u>

$I = E+1, \; R = I+2, \; 3 \le E \le 8$

Three cases to consider:

$E = 3, I = 4, R = 6$ ×

$E = 4, I = 5, R = 7$ ✓

$E = 5, I = 6, R = 8$ ×

A	M	Y	S	E	I	R	F	T	C
1	0	2	9	4	5	7	*	~~*~~	*

$F+T = E+10 \Rightarrow F+T = 14$

$T+E = C+9 \Rightarrow T+4 = C+9$

$\Rightarrow T = C+5$

$\Rightarrow C \le 4$

$C \neq 0, 1, 2$ or $4 \quad \therefore C = 3$

$\therefore T = 8$ and $F = 14 - 8 = 6$

<u>SOL'N</u>:

$$\begin{array}{r}
65682 \\
+\;981849 \\
\hline
1047531
\end{array}$$

$$\begin{array}{rcccccc} & & S & E & V & E & N \\ & & S & E & V & E & N \\ + & & & & S & I & X \\ \hline & T & W & E & N & T & Y \end{array}$$

1) $C_6 = 1 \therefore T = 1$

2) Column 4 EEE implies $E = 0, 8$ or 9 depending on $C_4 = 0, 2$ or 1, respectively

3) Case 1: $E = 0$ ($C_4 = 0$)

$E = 0 \Rightarrow C_5 = 0$ and $\boxed{2S = W + 10}$

$\therefore$ W is even, $W \neq 0$

column 2: $C_2 + I = 1$ impossible

$\therefore C_2 + I = 1 + 10$ and $C_3 = 1$

$C_2 + I = 11 \Rightarrow C_2 = 2, I = 9$

column 3: $C_3 = 1$

$\therefore \boxed{2V + S + 1 = N}$ ($C_4 = 0$)

Now, $2S = W + 10 \Rightarrow S \geq 6$

But, $2V + S + 1 = N \Rightarrow 2V \leq 2$

$\therefore V = 0$ or 1 impossible

$\therefore E \neq 0$

4) Case 2: $E = 8$ ($C_4 = 2$)

$$\begin{array}{rcccccc} & 1 & 1 & 2 & & & \\ & & S & E & V & E & N \\ & & & 8 & & 8 & \\ & & S & E & V & E & N \\ & & & 8 & & 8 & \\ & & & & S & I & X \\ + & & & & & & \\ \hline & T & W & E & N & T & Y \\ & 1 & & 8 & & 1 & \end{array}$$

Column 2:

$C_2 + 16 + I = 1$ impossible

$C_2 + 16 + I = 11$ impossible

$\therefore C_2 + 16 + I = 21$ and $C_3 = 2$

so, $C_2 + I = 5$

$C_2 = 0 \Rightarrow I = 5$

$C_2 = 1 \Rightarrow I = 4$

$C_2 = 2 \Rightarrow I = 3$

Column 3: $2V + S + 2 = N + 20$

$\boxed{2V + S = N + 18}$

column 5: $2S + 1 = W + 10$

$\boxed{2S = W + 9}$, W odd

Generate and Test

$I = 5, 4$ or 3 ; W odd

E	T	I	W	S	V	N	X	Y
8	1	5	3	*	*	*	*	X

$W = 3$ and $2S = W + 9 \Rightarrow \boxed{S = 6}$

$I = 5 \Rightarrow C_2 = 0 \Rightarrow 2N + X = Y$

$\Rightarrow \boxed{2N = X - Y}$ and $X - Y$ is even

$\therefore X - Y = 4$ or 8 (only possibilities)

$X - Y = 4 \Rightarrow X = 4$ or $Y = 0$ (only possibilities)

$X - Y = 8$ (not possible)

$\therefore \boxed{X = 4}$ and $\boxed{Y = 0}$

$X = 4, Y = 0$ and $2N = X - Y \Rightarrow \boxed{N = 2}$

$N = 2, S = 6$ and $2V + S = N + 18$

$\Rightarrow \boxed{V = 7}$

We have a sol'n !!!

To prove the sol'n is unique you need to examine the remaining cases:

$$E = 8 \begin{cases} I = 5, \ W = 7 \text{ or } 9 \\ I = 4, \ W \text{ odd} ; \ I = 3, \ W \text{ odd} \end{cases}$$

$E = 9$ and discard.

SOL'N:

$$\begin{array}{rr} & 68782 \\ & 68782 \\ + & 650 \\ \hline & 138214 \end{array}$$